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RESTORATION OF TIME FUNCTIONS DISTORTED BY TRANSDUCERS DESCRIBED BY DIFFERENTIAL EQUATIONS

Heinz H. Grote

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UNITED STATES ARMY

ELECTRONICS RESEARCH AND DEVELOPMENT LABORATORY

FORT MONMOUTH, N.J.

U. S. ARMY ELECTRONICS RESEARCH AND DEVELOPMENT LABORATORY

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RESTORATION OF TIME FUNCTIONS DISTORTED BY TRANSDUCERS

DESCRIBED BY DIFFERENTIAL EQUATIONS

Heinz H. Grote

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Abstract

A method for the restoration of time functions which have passed through a transducer described by one or more differential equations is presented. The restoration of signals which have passed through a low-pass filter is demonstrated and shows surprising similarity between the original and the restored time functions. This method is especially suited for application on the analog computer.

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RESTORATION OF TIME FUNCTIONS DISTORTED BY TRANSDUCERS
DESCRIBED BY DIFFERENTIAL EQUATIONS

OBJECTIVE

All transducers for which the relation between output and input is described by a differential equation yield an output which is not an analog replica of the input. The output is not simply determined by the input at the same instant, but depends on the time history of the input.

Many efforts have been made to recover the input time function for its measured output function; but in spite of considerable computational effort, only moderate approximation is obtained.

A simple method for the input restoration, recently proposed and implemented by the author, is discussed in this report.

DISCUSSION

Definitions

A transducer will be considered in this discussion as a "black box" which yields an output $o(t)$ when an input $i(t)$ is applied. The black box comprises the entire transducing system, including the coupling of the input member to the medium of which a parameter will be measured and the loading of the output member. Output and input are functions of time, and in general have different dimensions. With the exception of the theoretical case that the transducer has an infinitely wide frequency-band characteristic, the relation between output and input is described by a differential equation containing the input and its time derivatives and the output and its time derivatives. This equation will be referred to as "transfer equation."

For the proposed method it is not necessary that the transfer equation be available in closed standardized form, with separated terms of input and output. It is sufficient if the black box is described by one or more differential equations, with each equation containing the same or other transducer parameters so long as input and output are among them.

The black box is constant with time, space, and other parameters, so that an ordinary differential equation is sufficient to describe it. Black boxes that vary with a known parameter yield to the same techniques as long as they can be described by ordinary differential equations, with coefficients varying so slowly that all transients have died out before the coefficient changes markedly. In case the transfer equation is a linear differential equation, it can be reduced to an ordinary complex algebraic equation in the frequency domain, with the frequency ω as the only variable. The black box is then described by the ratio of output $O(\omega)$ over input $I(\omega)$, which is defined as the "transfer" function $G(\omega)$.

For the case where not only input and output have to be compared, but other parameters such as, for instance, input loading, are involved, the "transfer matrix" is available. This is a mathematical shorthand expression for a system of linear differential equations describing in a compact form several qualities of the black box. The transfer matrix has the advantage that it can be assembled in a form that allows the study of the influence of changes in particular elements of the transducer. It may be composed of a group of matrices, each describing only part of the transducer. The individual matrix may then be independent of its neighboring matrices.

Present Methods

The methods of input restoration known to the author make use of the fact that, for linear differential equations as transfer equations, the input time function can be described by the sum of sinusoids of time-constant amplitude, with each sinusoid having its characteristic amplitude, frequency, and phase (Fourier transform). For each sinusoid the output $O(\omega_k)$ can then be represented as the product of the sinusoidal input $I(\omega_k)$ of frequency ω and the complex transfer characteristic $G(\omega_k)$.

$$O(\omega_k) = I(\omega_k) \cdot G(\omega_k).$$

If this equation is written in the form

$$I(\omega_k) = \frac{O(\omega_k)}{G(\omega_k)} ,$$

it represents the equation needed for input restoration. It cannot be solved, however, when $G(\omega_k)$ is zero while $O(\omega_k)$ still has a finite value. Then a complete restoration is not possible. An approximation can be attempted by filtering out the critical frequencies for which $G(\omega) = 0$, or by approximating the inverse transfer function $\frac{1}{G(\omega)}$ by a function which is not critical. The error is determined by the difference between the area underneath the approximation of $\frac{O(\omega)}{G(\omega)}$ and the area of $\int_0^\infty \frac{O(\omega)}{G(\omega)} \, d\omega$.

Instead of transforming the output function from the time domain to the frequency domain for each measurement and multiplying it with the inverse transfer function and transforming this back to the time domain, proposals have been made to perform this process only once for the transfer function, or an equivalent, and determine the input time function by convolving the output time function with the transformed transfer function equivalent. This is based on the theorem that for linear transducers the output can be obtained as the sum of individual outputs resulting from an infinite sequence of input Dirac pulses

$$o(t) = \int_{-\infty}^{t} i(\tau) \; g(t-\tau) \; d\tau.$$

In this equation, t is the instant of measurement, τ is the instant the Dirac pulse is applied, and $g(t-\tau)$ is the response of the black box to a unit Dirac pulse. This convolution integral allows computation of the output of a black box from the input-time function without taking refuge in the frequency domain.

In order to now apply the convolution for the reverse operation of computing the input for a given output, the reverse of $g(t-\tau)$ has to be determined. The task now is to find an input function h, which generates a Dirac pulse at the output of the black box, because the aim is to determine the restored input by means of the convolution

$$i(t) = \int_{-\infty}^{t} o(\tau)\ h(t-\tau)\ d\tau,$$

where $h(t-\tau)$ is the above-described inverse pulse-response function. Since a Dirac pulse is equivalent to the sum of all frequencies from zero to infinity with equal amplitude, it is evident that no function h can be found that strictly satisfies the requirement. Approximations have been determined for some transfer functions. Examples of restoration attempts (see, for instance, Ref. 1) yield only moderate results.

In summing up it can be stated that, in spite of considerable computational efforts, the methods sketched above yield only moderate results.

Proposed Method

Instead of utilizing the transfer function which requires the detour over the frequency domain and which is applicable only when the transfer equation is a linear differential equation, it will be shown that it is possible to stay with the differential equation in the time domain.

In the transfer equation the input to the transducer is the forcing function of the homogeneous differential equation, and the task of finding a mathematical solution is to determine that particular output function which satisfies the differential equation. The digital computer solves this problem by a sequence of iterations, and the analog computer by a feedback system, which is in essence a continuously operating iteration circuit. If, however, the output function is given, insertion of the output function with all its required derivatives in the transfer equation yields directly the input function with its derivatives. The next step, then, is to solve the differential equation containing the unknown input function and its derivatives and the above computed function containing the measured output and its derivatives as forcing function.

The only limitation of the above procedure is the limit of accuracy with which the derivatives can be determined. Quantizing of the output function distorts the derivatives, and therefore digital computers cannot easily be used for the proposed restoration method. It is, however, ideally suited for analog computation so long as the derivatives of the output function are not distorted by noise inherent to the output signal.

Noise which might enter the computer during the restoration is reduced by arranging the circuitry so that differentiation circuits are used only if a substitution by integrators is not possible. The response of the differentiation circuits is then limited at higher frequencies which are generally above the region of interest.

A typical example of how differentiation may be avoided in analog computation is demonstrated for the following differential equations:

$$Ai + B\frac{di}{dt} = Co + D\frac{do}{dt} + E\frac{d^2o}{dt^2} + F\frac{d^3o}{dt^3}.$$

Integration of this equation yields

$$A\int i\,dt + Bi = C\int o\,dt + Do + E\frac{do}{dt} + F\frac{d^2o}{dt^2}.$$

This is rearranged:

$$-F\frac{d^2o}{dt^2} = E\frac{do}{dt} + Do + C\int o\,dt - Bi - A\int i\,dt.$$

Assuming that $\frac{d^2o}{dt^2}$ is available, its integration yields $\frac{do}{dt}$, and further integration o and $\int o\,dt$. The flow pattern is indicated by the integration signs and the arrows underneath the variables.

Another integration chain is obtain from the input i, and leads to $\int i\,dt$. After applying the proper factors and summing up all parts on the right-hand side, $F\frac{d^2o}{dt^2}$ is determined. Thus, the feedback loop is closed, and finally o is read out. Since, in general, transfer functions contain higher order derivatives of the output than of the input, some differentiation cannot be avoided in the process of restoring the input function.

For a known output, with the input to be determined, the above equation may be written as follows:

$$A\int i\,dt + Bi = C\int o\,dt + Do + E\frac{do}{dt} + F\frac{d^2o}{dt^2}.$$

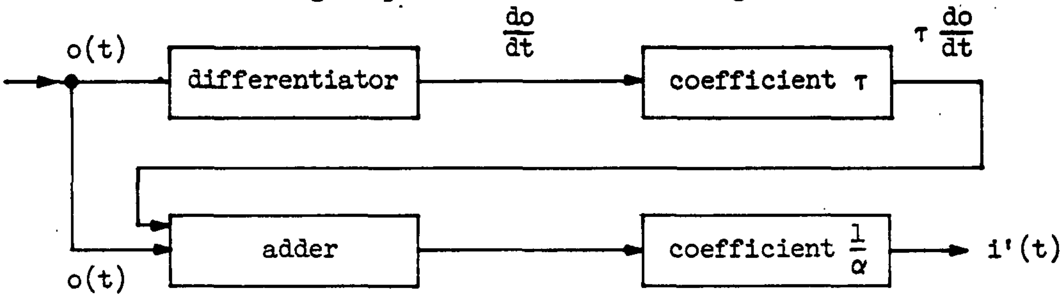
Here, the sign d denotes a differentiation. Two differentiations have to be performed in the above example. (The number of differentiations is equal to the difference in the degree of the differentials of output minus input.) Two analog differentiators are treated in appendix I.

Restoration Examples

Some examples of input function restoration on the analog computer are demonstrated below. One of the simplest transfer equations is

$$a \cdot i(t) = O(t) + \tau\frac{do}{dt},$$

where a is a dimensional factor, $i(t)$ the input function of time, $o(t)$ the output function of time, and τ a time constant. This is the differential equation of a low-pass filter, a first-order linear transducer. With the output o(t) measured, the task is to restore the input i'(t). This can be achieved on the analog computer with the following circuit:



To check the degree of restoration, a function i(t) was passed through a low-pass filter simulated on the computer. The output o(t) of the filter was recorded together with the original input function i(t) and the restored input function i'(t).

Figure 1 shows the restoration of a square wave. After passing through the low-pass filter, it is distorted to the known functions $1 - e^{-\frac{t}{\tau}}$ for the rising part and $e^{-\frac{t}{\tau}}$ for the decaying part. If the step persists long enough, saturation occurs; otherwise, decay occurs before the maximum is reached. After passing through the restoration circuit, a perfect square wave is obtained again.

In Fig. 2 the restoration of staircase steps is demonstrated. Even though the output curve shows only slight bends, the restored curve exhibits sharp edges; and even the minutely sharp transients in switching from one level to another are precisely restored.

In Fig. 3 a rapid sequence of steps is treated. Minute differences in the reproduction of the edges are due to the response of the recording servo and are not caused by the restoration circuit.

A very convincing restoration is demonstrated in Fig. 4. Generally, the sharp peaks are very sensitive to distortion, but it can be seen that here also the frequency response of the differentiators was high enough without introducing noise.

It is pointed out that in the previous examples a very simple transfer equation was implemented and that the signal-to-noise ratio was very favorable. Further investigations will show the influence of noise on the restoration ability, especially on higher order transfer functions. More complicated transfer equations will have to be investigated, and a restoration will have to be attempted for nonlinear transfer equations.

CONCLUSION

The excellent results obtained with the restoration of signals passed through a simple transducer encourage attempts to study higher order transfer

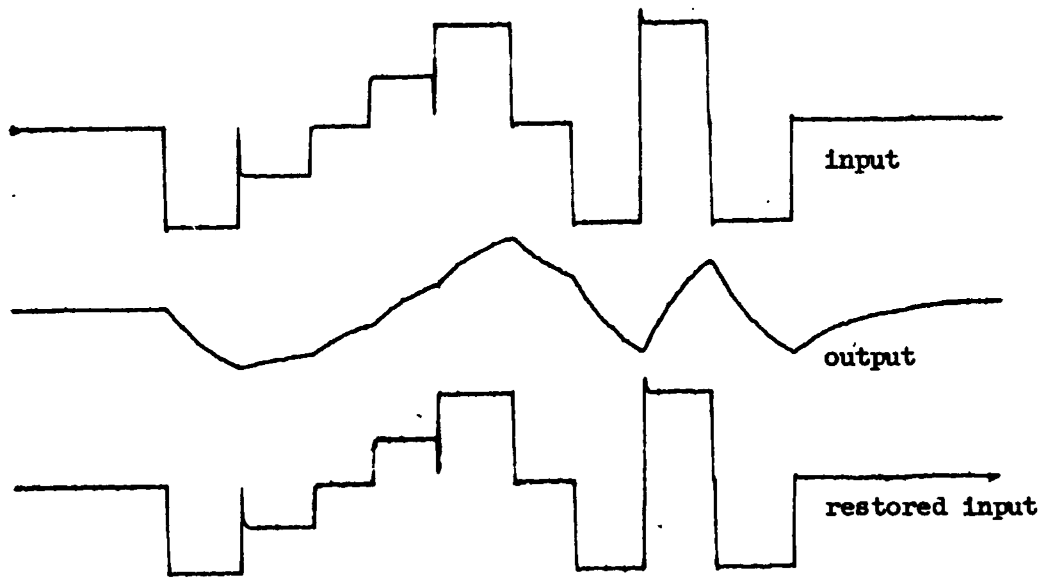


Fig. 1. Square Wave Restoration

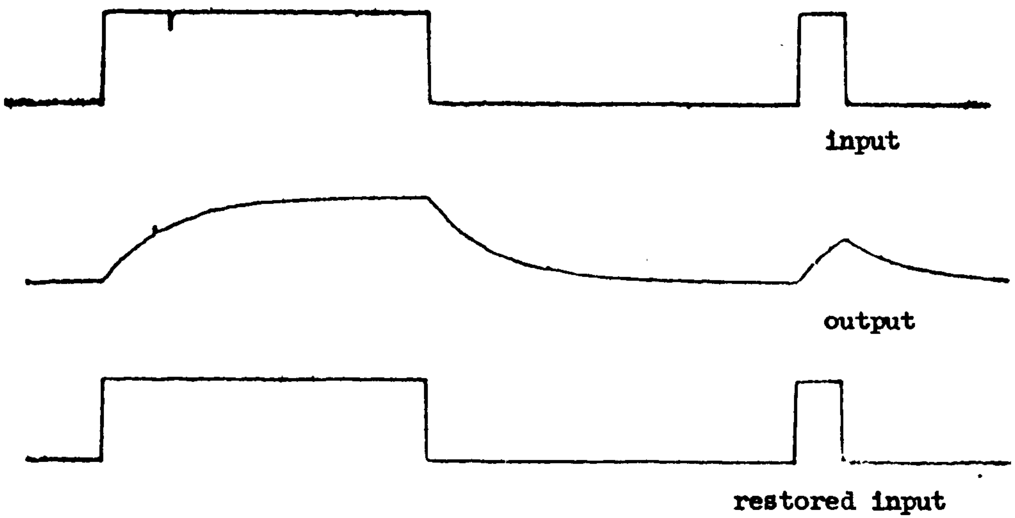


Fig. 2. Staircase Restoration

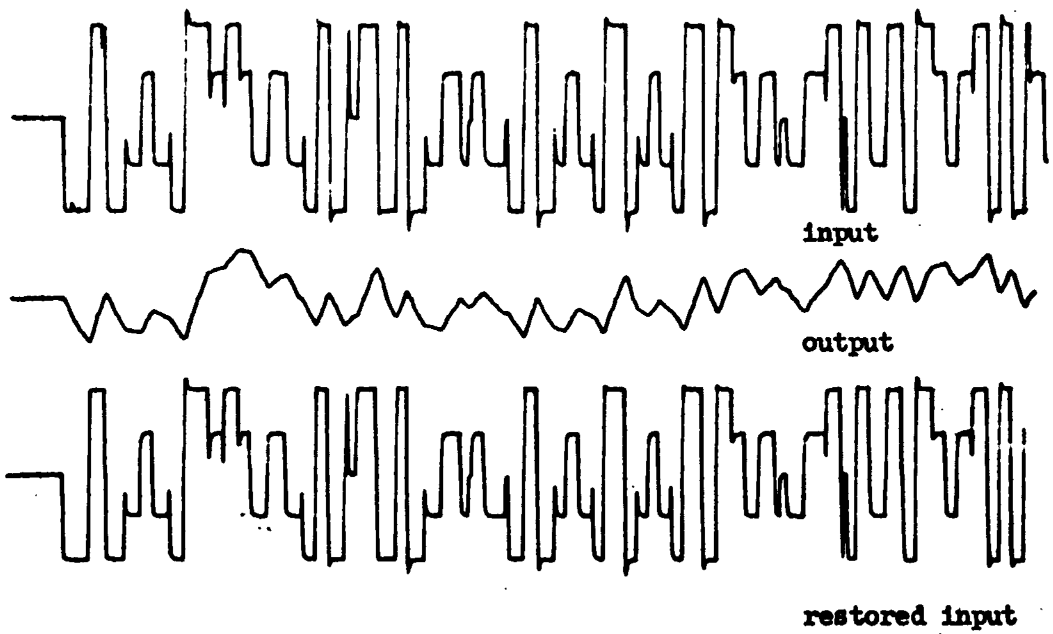


Fig. 3. Restoration of Sequence of Steps

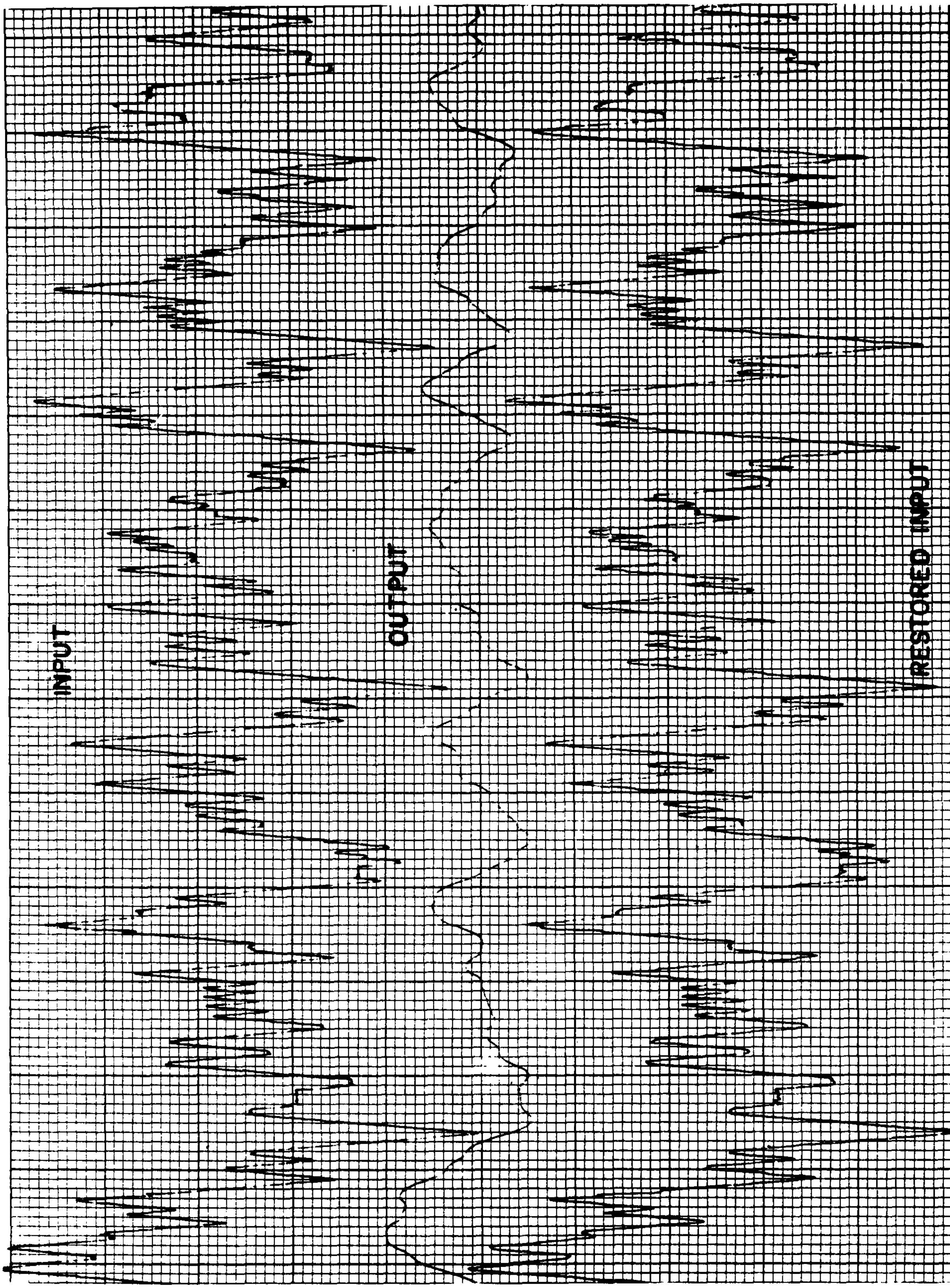


Fig. 4. Restoration of a Time Function

functions and also nonlinear transfer functions. Special attention will have to be paid to noise that is superimposed on the signal.

ACKNOWLEDGMENT

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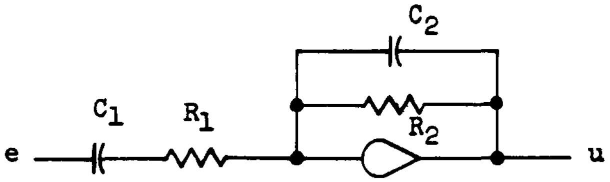
APPENDIX I

Analog Differentiation Circuits

the following the two most essential differentiation circuits pre- in the literature are discussed. One circuit is best suited for itted operational amplifiers or special-purpose arrangements. This t consists only of one operational amplifier with a capacitor-resistor and another capacitor-resistor feedback. The other circuit, consist- of two operational amplifiers and one integrator, is specially designed r general-purpose analog computers with committed input resistors and feedback resistors or capacitors.

The first differentiation circuit is shown in the following figure.



The transfer equation of this circuit is

$$u = -R_2C_1 \frac{de}{dt} - (R_1C_1 + R_2C_2) \frac{du}{dt} - R_1R_2C_1C_2 \frac{d^2u}{dt^2} .$$

For $R_1C_1 = R_2C_2 = \tau$, this becomes

$$u = - \frac{R_2}{R_1} \tau \frac{de}{dt} - 2\tau \frac{du}{dt} - \tau^2 \frac{d^2u}{dt^2} .$$

Resolving for e yields

$$u = -\frac{R_2}{R_1} \tau \frac{de}{dt} \left[1 - 2\tau \frac{de}{dt} + 3\tau^2 \frac{d^2e}{dt^2} - \ldots + (-1)^n (n+1) \tau^n \frac{d^ne}{dt^n}\right].$$

Thereby the relative error in determining the differential quotient $\frac{de}{dt}$ is

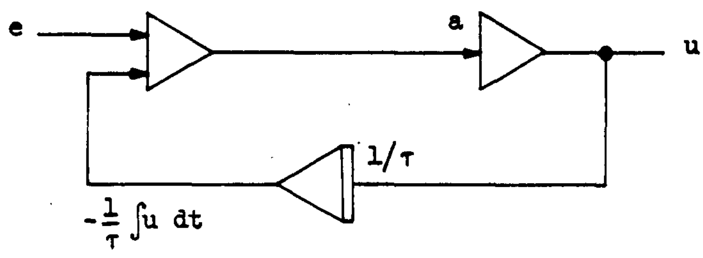
$$\frac{\Delta \frac{de}{dt}}{\frac{de}{dt}} = \epsilon = -2\tau \frac{de}{dt} + 3\tau^2 \frac{d^2e}{dt^2} - \ldots + (-1)^n (n+1) \tau^n \frac{d^ne}{dt^n} .$$

For a sinusoidal input signal $e = E \exp j\omega t$, the error ϵ is

$$\epsilon = -j2\tau\omega + j^2 3\tau^2\omega^2 - \ldots + j^n (-1)^n (n+1) \tau^n \omega^n .$$

If $\tau \ll \omega$, the relative error can be kept reasonably low.

The second differentiation circuit discussed looks quite different from the first, but it will be shown that the transfer equations of both are very similar.



$$u = \tau\frac{de}{dt} - \frac{\tau}{a}\frac{du}{dt}.$$

A comparison with the transfer equation of the previous circuit shows that they are equal for $C_2 = 0$ and $R_2/R_1 = a$ if $\tau = R_2C_1$.

APPENDIX II

Experiment Instrumentation

The experiments on an input restoration for which the results were presented in the body of this report were performed with the instrumentation described below.

A low-pass filter with the transfer equation

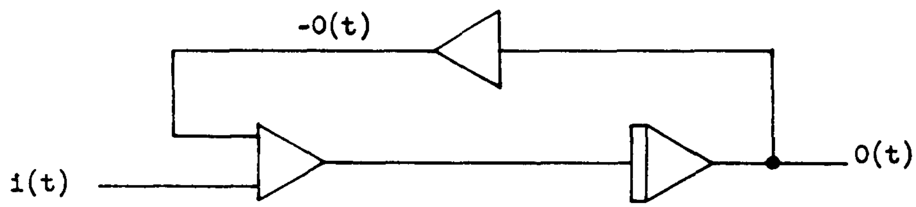
$$i(t) = o(t) + \tau \frac{do}{dt}$$

was simulated on the analog computer. Different signals $i(t)$ were passed through this simulation circuit and caused an output $o(t)$ which was then fed into the restoration circuit with an output $i'(t)$. All three functions--i, o, $i'(t)$--were accessible for recording. Because of the limitation of an x-y recorder to one plot at a time, arrangements had to be made to ensure phase accuracy for consecutive plots. Synchronization was obtained by recording all three functions together with a synchronization signal on an endless magnetic tape. The overall circuit diagram is shown in Fig. 5.

The test signal was generated by means of a ramp generator and a polarity reversing switch. Thereby, input signals in the form of zigzags with different amplitudes were obtained and recorded on magnetic tape. The tape was then played back.

The synchronization signal started the motion of the x-axis carriage of the plotter, and the signal passed through the filter simulation and the restoration circuit. A switch allowed the selection of the desired function.

The circuit of the filter simulation was



with the transfer equation

$$o(t) = \int \left[o(t) - i(t)\right] \, dt$$

$$i(t) = o(t) + \frac{do}{dt} .$$

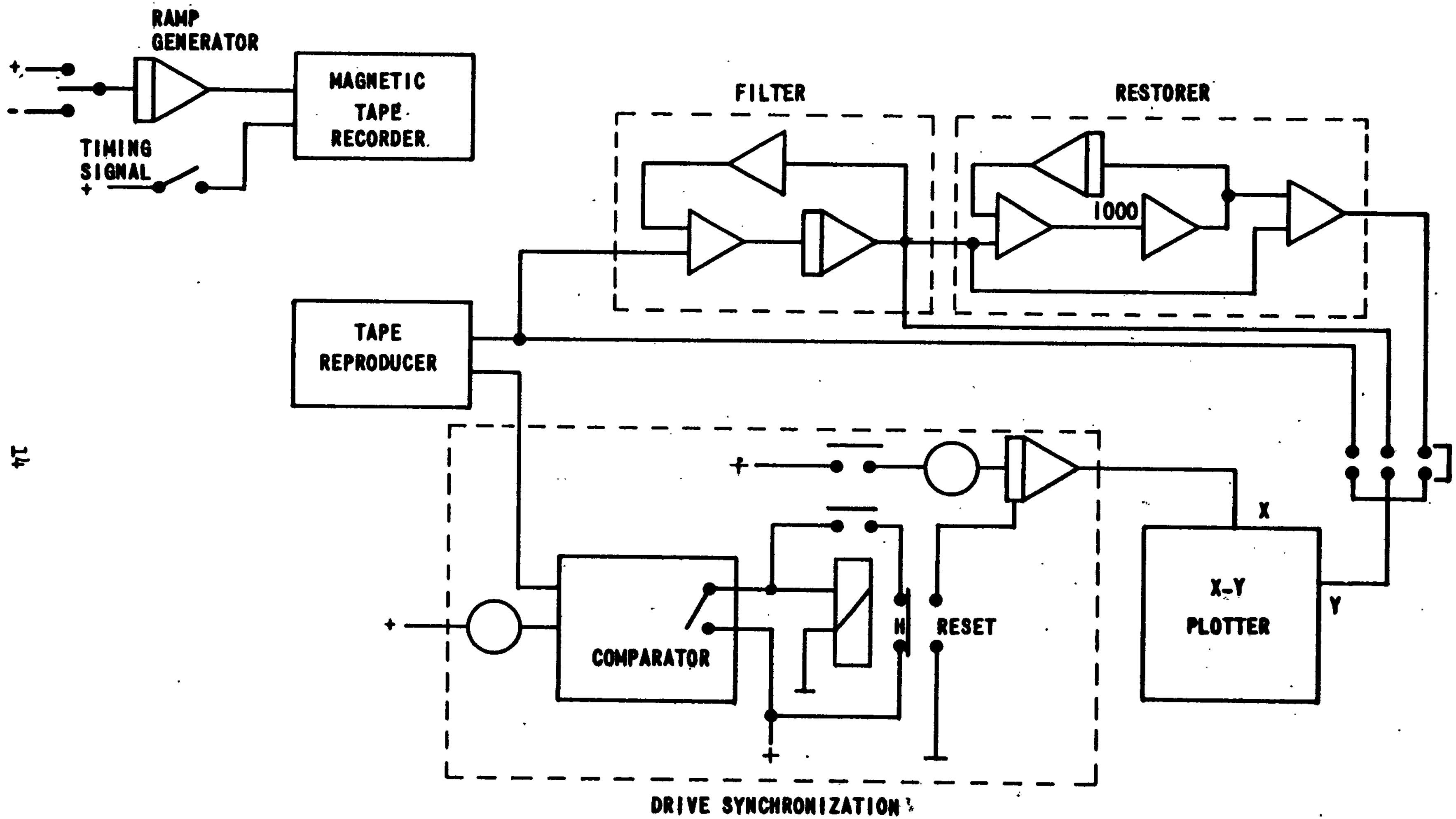
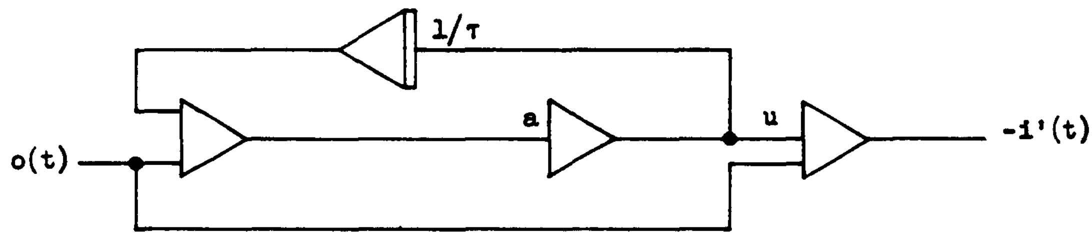


FIG. 5 TEST CIRCUIT FOR THE RESTORATION OF SIGNALS PASSED THROUGH A LOW-PASS FILTER

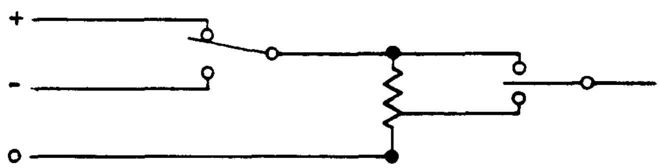
The circuit for the restoration of the input signal was



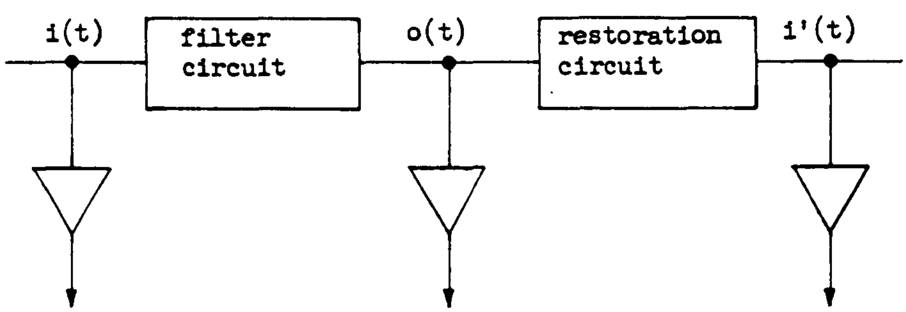
This circuit made use of the second differentiation circuit described in appendix I. It had the transfer equation

$$i'(t) = o(t) + \tau \frac{do}{dt} - \frac{\tau}{a} \frac{du}{dt}.$$

The accessories for the demonstration of the restoration of square wave were similar. The input function i(t) was obtained by changing the position of two switches which were arranged in the following manner:



The first switch changed the polarity of the signal, while the second switch allowed the choice of full voltage, no voltage, or half voltage. These signals were fed to the filter simulation and the restoration circuits. Input i(t), output o(t), and restored input i'(t) were fed over isolation amplifiers to three input circuits of the magnetic tape recorder.



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ATTN: Mr. Curtis H. Johnson
933 North LaBrea Avenue
Los Angeles 38, California

Commanding General 1
Desert Test Center
Fort Douglas, Utah

Commander 1
Air Force Cambridge Research
Laboratories, ATTN: CRZW
1065 Main Street
Waltham, Massachusetts

President 1
U. S. Army Arctic Test Board
Fort Greely, Alaska

Hqs, U. S. Army Test and 1
Evaluation Command
ATTN: AMSTE-EL
Aberdeen Proving Ground, Maryland

Hqs, U. S. Army Test and 1
Evaluation Command
ATTN: AMSTE-BAF
Aberdeen Proving Ground
Maryland

Hqs, U. S. Army Missile Command 1
ATTN: AMSMI-RB
Redstone Arsenal, Alabama

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ATTN: Director of Development Support
Army Chemical Center, Maryland

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Army Electronics Research and Development Laboratory, Fort Monmouth, N. J.

RESTORATION OF TIME FUNCTIONS DISTORTED BY TRANSDUCERS DESCRIBED BY DIFFERENTIAL EQUATIONS, by Heinz H. Grote, March 1963, 15 p. incl. illus., 9 refs. (AELRDL Technical Report 2337) (DA Task 3A99-27-005-02) Unclassified Report

A method for the restoration of time functions which have passed through a transducer described by one or more differential equations is presented. The restoration of signals which have passed through a low-pass filter is demonstrated and shows surprising similarity between the original and the restored time functions. This method is especially suited for application on the analog computer.

1. Signal Restoration
2. Transducers
3. Filters

I. Grote, Heinz H.
II. Army Electronics Research and Development Laboratory, Fort Monmouth, N. J.
III. DA Task 3A99-27-005-02

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